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THE ACOUSTICS OF A LARGE SPACE WITH A REPETITIVE PATTERN OF COUPLED ROOMS

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The acoustics are discussed of a large building divided by arches and columns into 77 coupled "subspaces" or rooms. Two models of coupled rooms have been devised to predict acoustical characteristics such as the decay of sound energy density and reverberation time. The first is a general model which can be applied to any system of coupled rooms. For each room of the 77 room system, the sound energy balance equation has been derived. The initial value problem, formulated by the system of 77 sound energy balance equations and initial steady state conditions, has been reduced to the eigenvalue problem, which has been solved numerically. The numerical results give fair agreement with the experimental results. The second model is approximate and could be derived because the particular building studied has a repetitive architectural pattern in which three repeatable rooms are identified. Consequently, with the approximate approach only three sound balance equations are required. The results obtained from the approximate model are in good agreement with results from the general model, as well as with the experimental results, for a substantial part of the space, except for the periphery.

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1. INTRODUCTION

Very large internal spaces have special problems associated with their size and shape. Large factory halls may be long and low with many small fittings and, as a result, there is a significant effect on both the reverberation time and the way in which the sound pressure level decreases with distance [1, 2]. Large churches may have long reverberation times because of their shape and the materials used in construction [3, 4]. In general, the acoustics of extensive internal spaces have always been difficult to analyze, because either the sound field within the space may not be diffuse or it may be only locally diffuse. In the latter case the assumption of the uniformity of sound energy density throughout the whole space of the large building would not be justified.

Large churches or cathedrals generally have heights which are of the same order as their widths. Although in a cathedral there may be a choir, transepts and a nave, divided by arches into several bays, the overall impression formed by an observer situated at, say, the crossing is in most cases of a single space. The subject of this paper is the behaviour of sound within a large public building in which the space is divided in an unusual way. In this building there are many arches and the height of the interior is relatively small compared with the length and width. Consequently, the space is dominated and divided

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by the columns and arches. The existence of this type of division enables the analysis of the acoustics of such a space to be achieved by a model of coupled rooms.

The internal space is separated into "subspaces" or "rooms" by the arches. In the acoustic model it is assumed that the sound field within a particular subspace is diffuse and is characterized by constant sound energy density. The subspaces, which are separated by arches, can be considered as a series of coupled rooms in which the flux of sound energy from one room to the next is taken into account. An energy balance equation is specified for each particular subspace of the building.

In the case of the building under consideration, the specific symmetry of the architecture of the internal space enables the system of equations to be reduced to a set of only three equations. This type of approximate approach considerably simplifies the process of acoustic analysis of large buildings with a repeated architectural pattern.

Thus, two possible models of coupled rooms are analyzed in this paper; a general model for any system of coupled rooms, and an approximate model which applies to spaces with a repeatable architectural pattern, such as is found in the building under consideration—the results obtained from the two models are compared.

The transfer of sound energy between coupled rooms was first studied by Buckingham [5] in relation to the transfer of energy from one room to another via a panel or window. Buckingham's work formed the basis for the method used in the measurement of sound reduction indices for panels or test components such as windows, etc. [6, 7]. In this method the two coupled rooms are reverberation chambers, one the source room and the other the receiving room; the two rooms are coupled through the test component.

The sound energy flux between two rooms has been studied in detail by Cremer and Müller [8], both for the case in which the rooms are coupled to an open area and when they are coupled through a component such as a door or window. Cremer and Müller identified two extreme situations. The first case occurs when a room that would normally have a sound source is connected via a coupling area to a second room which has an equivalent absorption area very much less than the coupling area. They describe this case as representing a high degree of coupling, in which the two rooms can be regarded as a single reverberant-type room. A low degree of coupling arises when the second room has an equivalent absorption area which is greater than the coupling area. In this case the coupling area between the rooms acts like an open window with respect to the source room. Thus, when the acoustics of the source room are considered, the open area to the second room acts like an open window, and an additional absorption area equal to the coupling area is added to the absorption area of the source room.

In the intermediate cases the rooms were described by Cremer and Müller as loosely coupled. If the absorption area of the first room (the sound source room) is much greater than that of the second room, it is possible for interesting effects to occur; namely, the sound decay curves exhibit "sagging" and "ballooned" appearances for sound source and non-source rooms, respectively. The sound pressure level change in the sound source room is rapid initially and much slower during the late stage of the sound decay. For the non-source room the sound level changes slowly at the beginning of the process and then progresses to a higher rate of decay. Kuttruff [9] has discussed the theory of coupled rooms, illustrated by an example of three coupled rooms in line.

The process of sound reverberation is analyzed in this paper, since it is an important factor in the estimation of speech intelligibility. In particular, the early stage of sound decay plays a significant role in sound perception [8–10]. The relationship between speech intelligibility and reverberation time has been discussed in many papers [11–15].

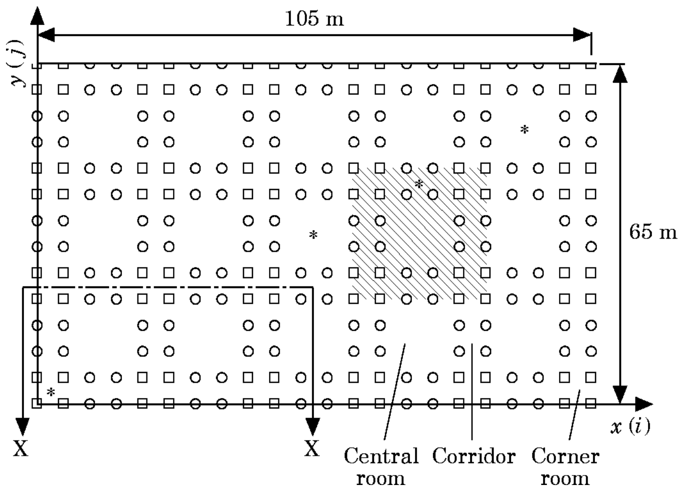

Figure 1. The plan of the building.

2. DESCRIPTION OF THE BUILDING

The interior of the building, to which the calculations and measurements apply, is shown in plan in Figure 1. The columns of the arches can be either circular or square, as shown on the plan. The columns and arches are shown more clearly on the elevation, shown in Figure 2, which is half the section through X–X in Figure 1.

The space considered has a height of 9·64 m, while its length is 105 m and width 65 m; it can be divided into 77 subspaces or rooms. Each room of the 77 room system is numerated by indices i and j along the x- and y-axes, respectively, as shown in Figure 1. This notation is used in the general model of coupled rooms; see section 3.1.

Three different types of rooms can be identified, as indicated in Figure 1; namely, the central rooms, the corridors and the corner rooms. Each central room is of dimensions 15 m by 15 m in plan, the corridors are 15 m long by 5 m wide and the corner rooms are 5 m by 5 m. There are three decorative arches, supported on columns, separating the central area from the corridor. The ceiling above the central area is deeply coffered and divided into nine sections.

The acoustic coupling between the rooms is relatively large. The average sound absorption coefficients in the rooms are similar; slightly greater in the central rooms at mid- and high frequencies because of the carpets on the floor. The coupling area between one side of a central room and a corridor is 84·6 m^2 and the coupling area between one side of a corner room and a corridor is 32·2 m^2. These coupling areas are quite large; almost 60% of the boundary area of a central room is open to adjacent rooms and available for the transmission of the sound.

The volume of a central room is 2060 m^3, the volume of a corridor is 683 m^3 and the volume of a corner room is 224 m^3. The total volume of the 77 rooms is 62 230 m^3.

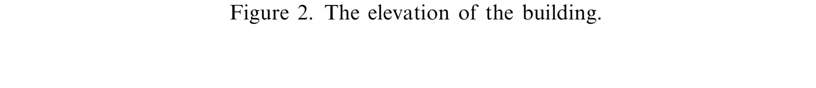

Figure 2. The elevation of the building.

TABLE 1

Sound absorption coefficients of materials used in the building

	Octave band centre frequency (Hz)					
	125	250	500	1 k	2 k	4 k
Marble	0·01	0·01	0·015	0·02	0·02	0·02
Plaster	0·12	0·10	0·06	0·04	0·04	0·03
Stone	0·10	0·10	0·10	0·10	0·10	0·15
Artificial stone	0·10	0·12	0·15	0·18	0·20	0·20
Carpet	0·09	0·08	0·21	0·32	0·45	0·50

The space considered (see Figure 1) is bounded by a solid wall at $y = 65$ m and by arches and columns at $y = 0$, $x = 0$ and $x = 105$ m.

The materials used for the interior building are mostly marble, stone and two types of plaster, one ordinary and the other referred to as artificial stone. The floor and the columns at the edge of the central rooms are of marble. The four columns of the corner rooms are of stone. The ceiling, the arches and all decorative work is of artificial stone. In the numerical calculations presented here, it is assumed that carpets are laid on the floor in the central rooms. In general, the absorption of sound by the carpets is greater than, for example, that of the ceiling. Additionally, the absorption at the floor may change considerably due to a non-uniform distribution of people in different areas. The surface areas and volumes for the rooms were found from architectural drawings. Sound absorption coefficients were deduced from tables, for example [7, 16, 17], from manufacturers' catalogues and from the authors' measured data. The absorption coefficients used are listed in Table 1.

The values of the absorption areas for each room are listed in Table 2.

Additional sound absorption arises because of air absorption and is included in the values quoted in Table 2. The air absorption depends upon the room volume, frequency, temperature and relative humidity [18].

The building has a repetitive pattern, which can readily be appreciated from Figure 2. In Figure 1, where the plan of the building is presented, a subsystem of nine rooms is shown hatched. These nine rooms are one central room four corridors and four corner rooms. The whole building interior can be regarded as a system of overlapping nine room subsystems.

The approximate theory of coupled rooms (see section 3.2) makes use of the repetitive nature of the architectural pattern of the interior, generated by the overlapping subsystems

TABLE 2

Equivalent absorption areas (m^2) *at different octave band centre frequencies for the three types of rooms* (*including air absorption*)

	Octave band centre frequency (Hz)					
	125	250	500	1 k	2 k	4 k
Central rooms	79·7	80·1	114·1	152·7	201·6	233·0
Corridors	26·4	28·0	30·7	36·6	45·6	52·6
Corner rooms	12·7	13·1	13·7	15·5	18·3	19·1

of rooms. In the approximate model the central rooms are referred to as rooms 1, the corridors as rooms 2 and the corner rooms as rooms 3.

3. THEORY

3.1. MODEL OF COUPLED ROOMS: GENERAL APPROACH

The interior of the large building is treated as a system of 77 coupled rooms, as is shown in Figure 1. It is assumed that in each individual room the sound field is diffuse and characterized by a different value of the time-averaged sound energy density. The non-stationary process of the reverberant sound energy decay is considered. Initial conditions at time $t = 0$, when the sound sources are switched off, are assumed to be steady state.

The sound energy balance equations, which describe the decay of the sound energies in the rooms, take into account the sound energy absorbed in every rooms, as well as the sound energy transfer between coupled rooms. Hence for the system considered of the coupled rooms the sound energy balance equations form a set of 77 linear differential equations of first order:

$$V_{i,j}\,(\mathrm{d}E_{i,j}/\mathrm{d}t) = -cA_{i,j}\,E_{i,j}/4 + cS_{i-1,i}\,[f(i;1)E_{i-1,j} - E_{i,j}]/4 + cS_{j-1,j}\,[f(j;1)E_{i,j-1} - E_{i,j}]/4 + cS_{i+1,i}\,[f(i;k)E_{i+1,j} - E_{i,j}]/4 + cS_{j+1,j}f(j;n)\,[E_{i,j+1} - E_{i,j}]/4, \quad (1)$$

where $i = 1, \ldots, k\,(=11)$ and $j = 1, \ldots, n\,(=7)$. The speed of sound is c, $V_{i,j}$ is the volume of room (i, j), $A_{i,j}$ is the equivalent absorption area of room (i, j) (except the coupling area), $E_{i,j}$ is the sound energy density in room (i, j) and $S_{i-1,i}$, $S_{j-1,j}$, $S_{i+1,i}$ and $S_{j+1,j}$ denote the coupling areas between room (i, j) and all four adjacent rooms $(i-1, j)$, $(i, j-1)$, $(i+1, j)$ and $(i, j+1)$, respectively. The function $f(l; a)$, where l and a are integers, is introduced (see equation (1)) to incorporate boundary conditions: fully reflective wall at ($y = 65$ m) and coupling with open space ($x = 0$, $x = 105$ m, $y = 0$). The function $f(l; a)$ is defined as

$$f(l; a) = \mathrm{sgn}\,(|l - a|),$$

where sgn denotes the signum function.

The system of equations (1) can be presented in matrix (vector) form as

$$\mathrm{d}\{E\}/\mathrm{d}t = -[M]\{E\}, \quad (2)$$

where $[M]$ is a square matrix formed by the coefficients of equations (1) and $\{E\}$ is the vector $[E_{1,1} \ldots E_{i,j} \ldots E_{k,n}]^{\mathrm{T}}$. The sound energy density $E_{i,j}$ is the sth component of the vector $\{E\}$, as $s = n(i-1) + j$.

Since all eigenvalues λ_l of the matrix $[M]$ are distinct, the general solution of the system of equations (2) is of the form

$$\{E\} = \sum_{l=1}^{m} C_l \exp(-\lambda_l t)\{Y\}_l, \quad (3)$$

where $\{Y\}_l$ denotes the eigenvector belonging to the appropriate eigenvalues λ_l and where $l = 1, \ldots, m = kn$. The constants C_l are determined from the initial conditions (at $t = 0$). In the case considered, the initial conditions refer to steady state excitation.

3.2. REPETITIVE PATTERN OF COUPLED ROOMS: APPROXIMATE MODEL

In cases of large interiors with a repetitive pattern of coupled rooms, there is the possibility of applying an approximate but much more simple model for the coupled rooms. The approximation is achieved by neglecting boundary conditions. It is expected that the boundary conditions have a significant influence upon the acoustic properties of the periphery of the large building, but not upon its centre. Hence it seems to be justified to postulate that in large interiors with a repetitive pattern of coupled rooms the acoustics, at least of the central part of the space, is mainly controlled by the specific architectural pattern. In this case it can be assumed that the large space (see Figure 1) can be approximated by an infinite space, generated by the infinite number of overlapped subsystems of nine coupled rooms. The assumption of diffusivity of the sound field in individual rooms is maintained in the approximate model of the coupled rooms. Similarly, as in the general model, the non-stationary process of the reverberant sound energy decay is considered. Initial conditions at time $t = 0$, when the sound sources are switched off, are assumed to be steady state.

The system of sound energy balance equations, which describe the decay of the sound energies in the rooms, is reduced in this case to the three differential equations

$$V_1 (\mathrm{d}E_1/\mathrm{d}t) = -cA_1 E_1/4 + cS_{12}(E_2 - E_1), \tag{4a}$$

$$V_2 (\mathrm{d}E_2/\mathrm{d}t) = -cA_2 E_2/4 + cS_{12}(E_1 - E_2)/2 + cS_{23}(E_3 - E_2)/2, \tag{4b}$$

$$V_3 (\mathrm{d}E_3/\mathrm{d}t) = -cA_3 E_3/4 + cS_{23}(E_2 - E_3), \tag{4c}$$

where E_1, E_2 and E_3 are the sound energy densities in rooms 1, 2 and 3 (i.e., the central room, the corridor and the corner room), respectively (see section 2 for rooms notation in the approximate model); A_1, A_2 and A_3 are the equivalent absorption areas (excluding coupling areas) in rooms 1, 2 and 3; V_1, V_2 and V_3 are the volumes of rooms 1, 2 and 3, respectively; S_{12} is the coupling area between room 1 and room 2 and S_{23} is the coupling area between rooms 2 and 3. The coupling areas are the open areas between the rooms, i.e., the areas between the arches.

The particular solution of the system of equations (4a)–(4c) can be assumed to be of the form

$$\{E\} = \{E_0\} \exp(-\lambda t), \tag{5}$$

where $\{E\} = [E_1, E_2, E_3]^{\mathrm{T}}$, $\{E_0\} = \{E_{01}, E_{02}, E_{03}\}^{\mathrm{T}}$ and where E_{01}, E_{02}, E_{03} and λ are constants. Thus, after insertion of equations (5), the system of equations (4a)–(4c) becomes

$$([M] - \lambda[I])\{E_0\} = 0, \tag{6}$$

where $[I]$ is the unit matrix, the matrix $[M]$ is given by

$$[M] = \begin{bmatrix} 2\delta_1 & -cS_{12}/V_1 & 0 \\ -cS_{12}/(2V_2) & 2\delta_2 & -cS_{23}/(2V_2) \\ 0 & -cS_{23}/V_3 & 2\delta_3 \end{bmatrix}, \tag{7}$$

and δ_1, δ_2 and δ_3 are damping constants, defined as

$$\delta_1 = c(A_1 + 4S_{12})/(8V_1), \tag{8a}$$

$$\delta_2 = c(A_2 + 2S_{12} + 2S_{23})/(8V_2) \qquad \delta_3 = c(A_3 + 4S_{23})/(8V_3). \tag{8b, c}$$

The necessary and sufficient condition which allows the system of homogeneous equations (6) to have the solution $\{E_0\} \neq 0$ is

$$\det([M] - \lambda[I]) = 0. \quad (9)$$

The condition (9) leads to the characteristic equation

$$\lambda^3 - 2(\delta_1 + \delta_2 + \delta_3)\lambda^2 + \{4(\delta_1\delta_2 + \delta_1\delta_3 + \delta_2\delta_3) - (c^2/2)[S_{12}^2/(V_1V_2) + S_{23}^2/(V_2V_3)]\}\lambda + c^2[S_{12}^2\delta_3/(V_1V_2) + S_{23}^2\delta_1/(V_2V_3)] - 8\delta_1\delta_2\delta_3 = 0, \quad (10)$$

with the roots designated as λ_1, λ_2 and λ_3. The roots λ_1, λ_2 and λ_3 are characteristic eigenvalues for the eigenvalues problem represented by equation (6). The eigenvalues are functions of the damping constants δ_1, δ_2 and δ_3, as well as the room volumes and the coupling areas; see equation (10). The eigenvectors associated with the eigenvalues may be expressed as

$$\{Y\}_l = \begin{Bmatrix} 1 \\ V_1(2\delta_1 - \lambda_l)/(cS_{12}) \\ [2V_1V_2(2\delta_1 - \lambda_l)(2\delta_2 - \lambda_l) - c^2S_{12}^2]/(c^2S_{12}S_{23}) \end{Bmatrix}, \quad (11)$$

where $l = 1$, 2 and 3.

Thus the general solution of equation (4) for the sound energy density may be presented as a linear combination of particular solutions, namely

$$\{E\} = \sum_{l=1}^{3} C_l \exp(-\lambda_l t)\{Y\}_l. \quad (12)$$

The three constants C_1, C_2 and C_3 are determined from the initial conditions (at time $t = 0$) for the sound energy densities E_1, E_2 and E_3: i.e., they are expressed in terms of $E_1(0)$, $E_2(0)$ and $E_3(0)$ as

$$C_1 = \{[V_1V_2(\lambda_2 - \lambda_3)(2\delta_1 - \lambda_2)(2\delta_1 - \lambda_3) + (c^2S_{12}^2/2)(\lambda_2 - \lambda_3)]E_1(0) + cS_{12}V_2(\lambda_3 - \lambda_2)(2\delta_1 + 2\delta_2 - \lambda_2 - \lambda_3)E_2(0) + (c^2S_{12}S_{23}/2)(\lambda_2 - \lambda_3)E_3(0)\}/[V_1V_2(\lambda_3 - \lambda_1)(\lambda_2 - \lambda_3)(\lambda_2 - \lambda_1)], \quad (13a)$$

$$C_2 = [(2\delta_1 - \lambda_3)E_1(0) - cS_{12}E_2(0)/V_1 - C_1(\lambda_1 - \lambda_3)]/(\lambda_2 - \lambda_3), \quad (13b)$$

$$C_3 = [(\lambda_2 - 2\delta_1)E_1(0) + cS_{12}E_2(0)/V_1 + C_1(\lambda_1 - \lambda_2)]/(\lambda_2 - \lambda_3). \quad (13c)$$

The sound energy densities at time $t = 0$, $E_1(0)$, $E_2(0)$ and $E_3(0)$ refer to the steady state. They are functions of the sound powers P_1, P_2 and P_3 of the sound sources located in rooms 1, 2 and 3, respectively, as well as functions of the properties of the coupled rooms, such as the room volumes, the damping constants and the coupling areas:

$$E_1(0) = [(8V_2V_3\delta_2\delta_3 - c^2S_{23}^2)P_1 + 4cS_{12}V_3\delta_3P_2 + c^2S_{12}S_{23}P_3]/L, \quad (14a)$$

$$E_2(0) = [2cS_{12}V_3\delta_3P_1 + 8V_1V_3\delta_1\delta_3P_2 + 2cS_{23}V_1\delta_1P_3]/L, \quad (14b)$$

$$E_3(0) = [c^2S_{12}S_{23}P_1 + 4cS_{23}V_1\delta_1P_2 + (8V_1V_2\delta_1\delta_2 - c^2S_{12}^2)P_3]/L, \quad (14c)$$

where $L = 16V_1V_2V_3\delta_1\delta_2\delta_3 - 2c^2S_{23}^2V_1\delta_1 - 2c^2S_{12}^2V_3\delta_3$.

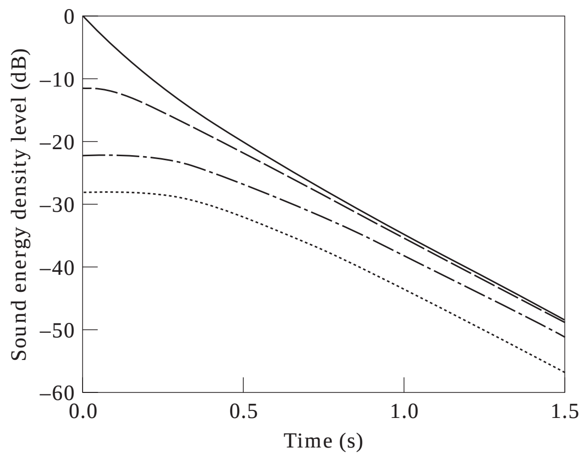

Figure 3. The decay of the sound energy density level for the octave band of centre frequency of 1000 Hz with the sound source in room $i = 6, j = 4$, for ——, room $i = 6, j = 4$; — — —, room $i = 8, j = 5$; –·–·–, room $i = 10, j = 6$; ····, room $i = 1, j = 1$.

4. RESULTS

4.1. THE GENERAL MODEL

The time decay of sound energy density has been analyzed for different frequencies at different locations in the interior. An example of the sound energy density decay with time is shown in Figure 3 for a frequency of 1000 Hz. The sound source was situated in the middle room ($i = 6, j = 4$) and switched off at $t = 0$ after steady state excitation. The sound energy density in room (i, j) is normalized with respect to the sound energy density in the middle room at $t = 0$ and is represented by the sound energy density level, which is defined as $10 \log [E_{i,j}(t)/E_{6,4}(0)]$. The rooms considered are a central room ($i = 6$, $j = 4$) in the middle of the 77 room system, a central room near the periphery ($i = 10, j = 6$), a corridor ($i = 8$, $j = 5$) and a corner room ($i = 1$, $j = 1$). These rooms are indicated by asterisks in Figure 1. The coupling effect is particularly noticeable in the early stage of the sound decay, where the sound energy decay curves have either a "sagging" (source room, $i = 6$, $j = 4$) or a "ballooned" (non-source rooms) appearance; see Figure 3.

The experimental results recorded in the sound source room are presented in Figure 4 in the form of curves of sound energy density level; a type of presentation which is possible, as the sound energy density is proportional to the mean-squared sound pressure [7]. The measurements were obtained with the aid of Brüel & Kjaer apparatus. A loudspeaker sound source (B & K type 4224) was placed in the central room in the middle of the space and the microphone of a sound level meter (B & K type 2231) was positioned about 10 m away in the same room. The sound source was switched off and the decrease in sound pressure level was recorded on a level recorder (B & K type 2317); see Figure 4. The reverberation time T was estimated from the approximate slope of the trace of sound pressure level against time, in accordance with the standard definition [8, 19].

The experimental data refer to frequencies of 500, 1000 and 2000 Hz—see Figures 4(a), 4(b) and 4(c), respectively—and are compared with theoretical results. The theoretical curves of the sound energy density level, presented in the form of continuous lines, result from combining the sound energy decay, after steady state excitation, with the background noise. Agreement between the experimental and theoretical results is good, apart from a short period of the late stage of the decay before the background noise becomes predominant.

The duration of the early part of the sound energy decay, when the decay curve slope is changing in time, is denoted by t_{eq}, the "equalization time". The equalization time elapses from $t = 0$, when the sound source is discontinued after steady state excitation, to the moment when the sound begins to decay with a constant rate. Finally, for a sufficiently long time $T_0 = \max\{t_{eq}(x, y)\}$ (where $0 < x < 105$ and $0 < y < 65$) the sound decays at the same constant rate for all locations in the space.

The contours of equalization time for the space considered are shown in Figure 5 for a frequency of 1000 Hz and for the sound source location in the middle room of the space. They indicate in which part of the space the effect of coupling is the most pronounced and how quickly the constant value of the sound decay rate is achieved in a particular location. The minimum of the equalization time is reached in an annular zone; see Figure 5. The effect of coupling, manifested in a longer equalization time, is most pronounced in the central part of the space and on the periphery, especially near the wall.

The examples illustrated in Figure 3, as well as the experimental results, provide evidence that the reverberation time as normally defined [8, 19] is not the appropriate characteristic for the early part of the sound energy decay. During the early stage of the process, the sound energy decay rate for each room changes considerably with time, as shown in Figure 6. In this figure the dependence of the slopes of the sound decay curves upon time is presented for two different rooms, for the source room ($i = 6$, $j = 4$) and an adjacent room ($i = 6$, $j = 5$). The slope represents a locally defined reverberation time T_r, which is

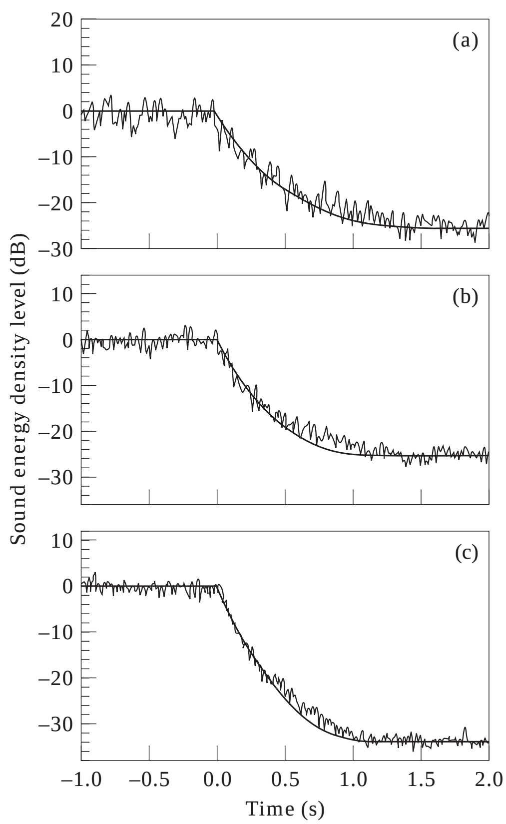

Figure 4. The experimental and theoretical decay of the sound energy density level with the source and receiver in room $i = 6$, $j = 4$ for octave band centre frequencies of (a) 500 Hz, (b) 1000 Hz and (c) 2000 Hz.

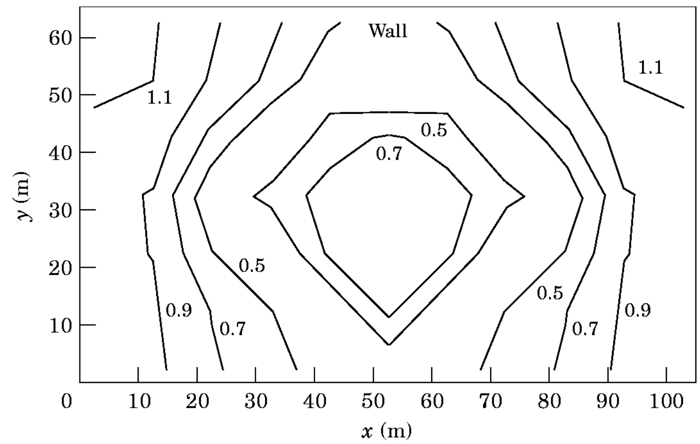

Figure 5. Contours of equalization time t_{eq} for the octave band of centre frequency 1000 Hz; sound source in middle room, $i = 6$, $j = 4$.

based on a 60 dB decay. After the elapse of the equalization time t_{eq}, the reverberation time T_r reaches a constant value T_l, which is defined in this paper as the late reverberation time $T_l = T_r$ ($t \geqslant t_{eq}$). The late reverberation time characterizes the sound decay during the late stage of the process and for the cases where coupling effects are negligible represents the normally defined reverberation time T [8, 19].

4.2. THE APPROXIMATE MODEL

The general method of coupled rooms discussed in section 4.1 can be applied to any space which is characterized by specific architectural properties that enable the space to be divided into groups of "subspaces" or rooms, each with a diffuse sound field. However, in the case of a large building with a repetitive architectural pattern an approximate method may be useful (see section 3.2), provided that the symmetry in the distribution of the sound sources is also maintained. Comparison of the sets of sound decay curves, solid and dashed curves, obtained from both the general and approximate models, respectively,

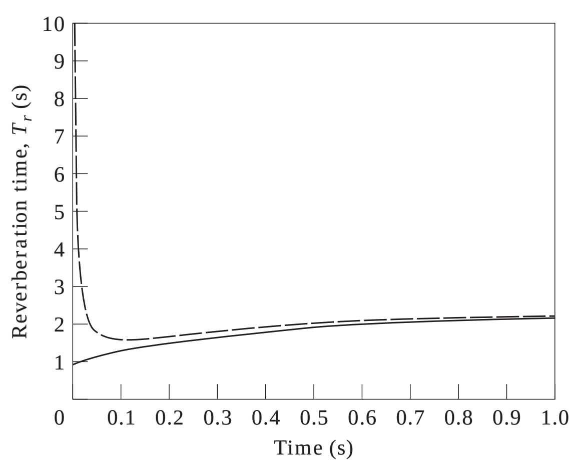

Figure 6. The reverberation time T_r as a function of time for the octave band of centre frequency 1000 Hz with the sound source in the middle central room ($i = 6$, $j = 4$) for ——, room $i=6$, $j = 4$; — — —, room $i = 6$, $j = 5$.

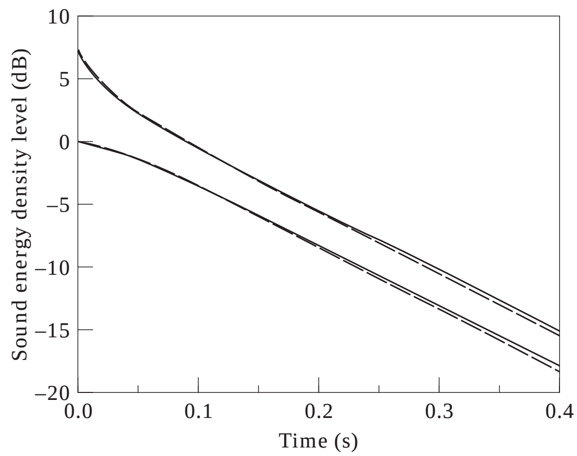

Figure 7. A comparison of the sound energy density decay curves for the general model and the approximate model for the octave band of centre frequency 500 Hz; people in all central rooms; sound source power ratios $P_1:P_2:P_3 = 1{:}1{:}5$ for the central rooms, the corridors and the corner rooms, respectively; sound decay in the corner room $i = 5$, $j = 3$ (upper curves) and the central room $i = 4$, $j = 4$ (lower curves). ——, General model; ———, approximate model.

for rooms in the middle part of the space is shown in Figure 7. The case shown in Figure 7 refers to a frequency of 500 Hz and to a higher than normal absorption area (500 m^2) in the central rooms. This amount of absorption arises when a large number of people are standing in the central rooms. Sound sources are distributed in all rooms with sound power ratios $P_1:P_2:P_3 = 1{:}1{:}5$ (subscripts refer to rooms 1, 2 and 3). Immediately after all of the sound sources are discontinued, the sound energy is fed from corner rooms to corridors and central rooms. As can be seen in Figure 7, the initial sound decay in the corner rooms is relatively steep, while the initial decay in the central rooms is gradual. Eventually, the decay rates are the same and equivalent to a reverberation time of 1·22 s.

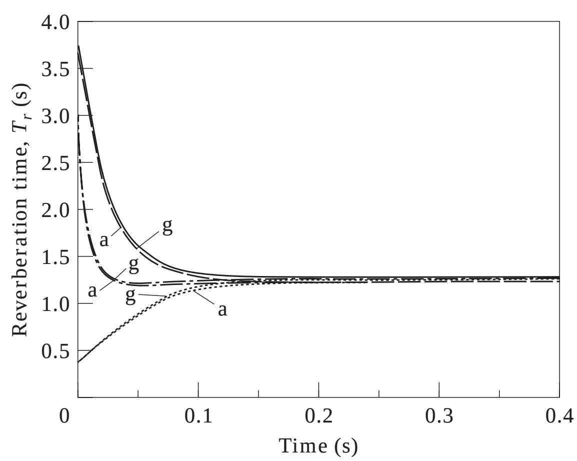

Figure 8. A comparison of the general and approximate models. The reverberation time T_r as a function of time for an octave band of centre frequency 500 Hz; people in all central rooms; sound sources in all rooms, with sound source power ratios $P_1:P_2:P_3 = 1{:}1{:}5$. ····, Corner room, $i = 5$, $j = 3$; –·–·–, corridor, $i = 6$, $j = 3$; ———, ——, central room, $i = 6$, $j = 4$; a, approximate model; g, general model.

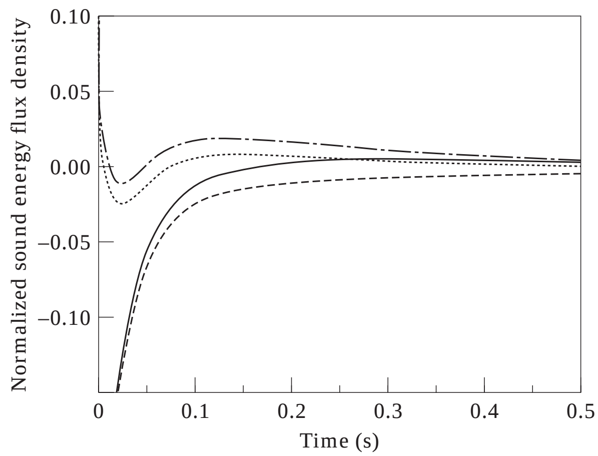

Figure 9. The normalized sound energy flux density as a function of time for an octave band centre frequency of 500 Hz; sound power ratios $P_1:P_2:P_3 = 1:10:1{\cdot}5$. General model: ——, $I_{6,3;6,4}$ flux density between rooms (6, 3) and (6, 4); –·–·–, $I_{5,3;6,3}$ flux density between rooms (5, 3) and (6, 3). Approximate model: — — —, J_{21} flux density between corridor and central room; ····, J_{32} flux density between corner room and corridor.

The change in time of the locally defined reverberation times T_r for both models and for different locations of the space is presented in Figure 8. Both models provide good agreement. The two different stages of the sound energy decay, the early with rapidly changing rate of sound decay and the late with a constant rate of sound decay, are clearly visible in Figure 8.

The early stage can be also characterized by a sound energy flux between the rooms. The sense and magnitude of the sound energy flux between the rooms depend upon the distribution of the sound sources and equivalent absorption area in the individual rooms. Shown in Figure 9 are the distributions in time of the normalized sound energy fluxes through the unit coupling surfaces between a corridor and a central room (dashed line) and between a corner room and a corridor (dotted line). The case considered in Figure 9 refers to a frequency of 500 Hz and sound sources with sound power relations $P_1:P_2:P_1 = 1:10:1{\cdot}5$. The equivalent absorption areas for the rooms are as listed in Table 2 (i.e., it refers to empty interiors without people). The densities of the normalized sound energy fluxes between a corridor (room 2) and a central room (room 1) and between a corner room (room 3) and a corridor (room 2) are defined as $J_{21}(t) = [E_1(t) - E_2(t)]/E_1(0)$ and $J_{32}(t) = [E_2(t) - E_3(t)]/E_1(0)$, respectively.

At the beginning of the early stage of the sound energy decay (see Figure 9), a corner room obtains energy from the corridors; later, the situation changes as energy flows from the corner room to the corridors; in the final stage of the early part of the sound energy decay a corner room gains energy from the corridors. The exchange of energy between the corridors and a central room has, in the case considered, a different character from the case of the sound energy transfer between corner rooms and corridors. The sound energy flows from the corridors to a central room and the magnitude of the sound energy flux diminishes monotonically with time.

The solid and dash–dot curves in Figure 9 are derived from the general model. They represent the normalized sound energy flux densities, as functions of time. The normalized sound energy flux densities are defined (see section 3.1 for the notation) as $I_{6,3;6,4}(t) = [E_{6,4}(t) - E_{6,3}(t)]/E_{6,4}(0)$ and $I_{5,3;6,3}(t) = [E_{6,3}(t) - E_{5,3}(t)]/E_{6,4}(0)$ and they refer to the sound energy exchange between rooms (6, 3) and (6, 4), i.e., between a corridor and a central room, and between rooms (5, 3) and (6, 3) (between a corner room and a corridor), respectively; see Figure 1. The results obtained from both the approximate and general models are in good quantitative and qualitative agreement.

5. DISCUSSION

The aim of this paper is to analyze the acoustics of large spaces with a specific architectural pattern which enables the whole space to be treated as a system of smaller subspaces. In a particular case when a large interior space is characterized by a repetitive architectural pattern the acoustics of the whole space can be represented by the local acoustical properties and the approximate, simple model can be introduced. It is also useful to know to what extent the approximate model can be used instead of the more complicated general model.

There are many architectural features which allow the space considered to be classified under Beranek's scheme [20] as highly diffusive. This raises expectations that the model assumption of diffuse sound fields in the coupled rooms is justified. Furthermore, the model of coupled rooms has provided good agreement with experiment; see Figure 4.

Although the couplings between an individual central room and the corridors and between the corridors and the corner rooms are strong, the resulting effect of the coupling of many rooms in the whole space is significant, especially for the early stage of the sound energy decay; see Figures 3 and 4. The duration of the early part of the sound energy decay depends not only on the properties of the space considered but also on the distribution of the sound sources throughout the space, as well as the ratio of the sound power of the sources. This early part of the sound decay, when the decay curve slope is changing in time, is characterized locally in space by the equalization time t_{eq} ; see Figure 5. More globally, for a group of rooms the time T_0 can be introduced, as the time required for a group of coupled rooms to reach the same rate of sound energy decay after the sound sources have been switched off. It should be noted that neither the definition of t_{eq} nor of T_0, as introduced in this paper, is the same as "early decay time" or "initial reverberation time" used in previous literature [8, 21]. More uniform distribution of sound sources leads to a shortening of the equalization time t_{eq} as well as T_0, as can be seen in Figure 10 for the case of three uniformly distributed sources. Figure 10 should be compared with Figure 5, which deals with the case of one source.

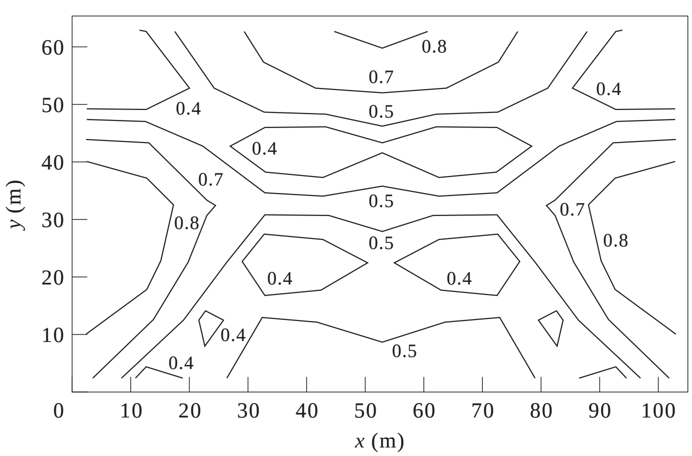

Figure 10. Contours of equalization time t_{eq} for the octave band of centre frequency 1000 Hz; sound sources in central rooms (2, 4), (6, 4) and (10, 4) with sound power ratios 1:1:1.

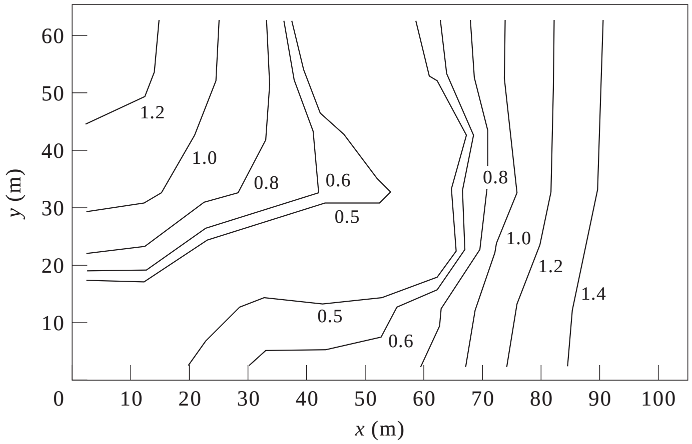

Figure 11. Contours of equalization time t_{eq} for the octave band of centre frequency 1000 Hz; sound sources in central rooms (2, 4), (6, 4) and (4, 6) with sound power ratios 1:1:1.

On the contrary, the asymmetrical distribution of the sound sources leads to an increase of the equalization time in a substantial part of the space, as is shown in Figure 11 for three sound sources located in the central rooms: room (2, 4), room (6, 4) and room (4, 6) with ratios of sound powers 1:1:1. The tendency for the equalization time to be prolonged is even more accentuated when the sound power ratios change to 1:1:5 for the rooms (2, 4), (6, 4) and (4, 6), respectively, and when the regularity of the pattern of the equivalent absorption areas is disturbed. These points are illustrated in Figure 12, which refers to the interior space, in which many people are grouped in two central rooms, (6, 4) and (4, 6); consequently, the values of the equivalent absorption areas are greater in these rooms than in the other central rooms of the space. The sound decay curves are shown in Figure 13 for the room conditions described in Figure 12. A more marked "sagging" shape of the sound decay curve for the middle room (with the sound source) indicates that the effect of coupling is more pronounced.

Since the approximate model gives good agreement with the general model for the middle section of the large space with a repetitive architectural pattern, it can be applied

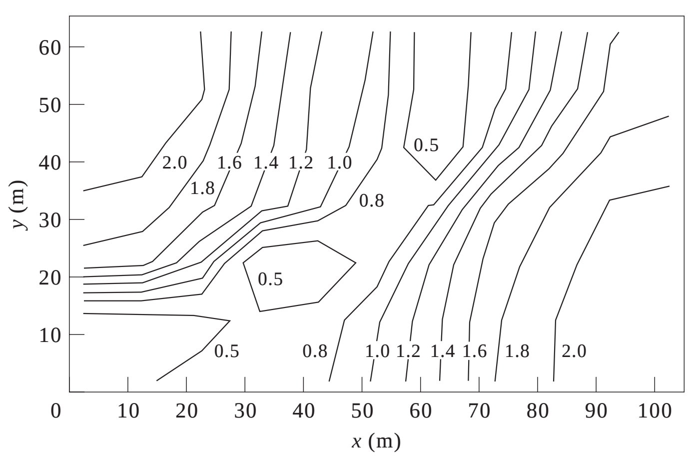

Figure 12. Contours of equalization time t_{eq} for the octave band of centre frequency 1000 Hz; sound sources in central rooms (2, 4), (6, 4) and (4, 6) with sound power ratios 1:1:5; people in central rooms (6, 4) and (4, 6).

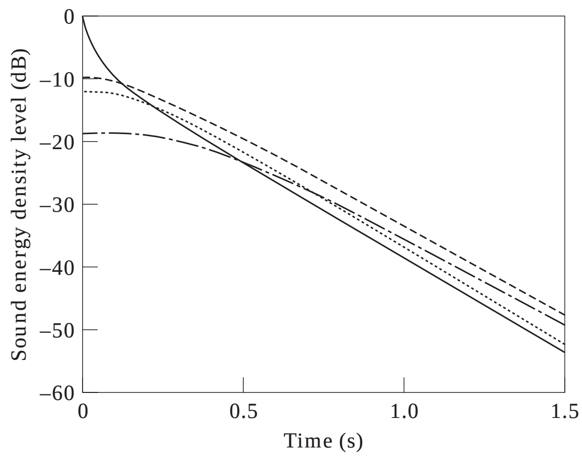

Figure 13. The decay of the sound energy density level for the case specified in Figure 12 for the octave band of centre frequency of 1000 Hz. ——, Room (6, 4); ———, room (8, 5), –·–·–, room (10, 6); ····, room (1, 1).

to estimate the optimal distribution of sound sources. The approximate model can be used, for example, to answer the question as to what kind of distribution of sound sources leads to a minimization of the coupling effects; i.e., to a reduction in the T_0 values. Analysis of the dependence of T_0 on P_1, the sound power of the sources in the central rooms, is shown in Figure 14 for frequencies of 125, 500 and 2000 Hz for three different values of P_2, the sound power of sources in the corridors (0, 1·25 and 5 W), and for $P_3 = 1$ W, where P_3 is the sound power in the corner rooms. The curves show that there are minimum values of T_0 for certain ratios of the sound powers of the sound sources.

There are further advantages of the approximate model apart from the simplicity already mentioned. The approximate model can be applied to estimate the late reverberation of any large space with a repeatable architectural pattern and with boundaries the properties

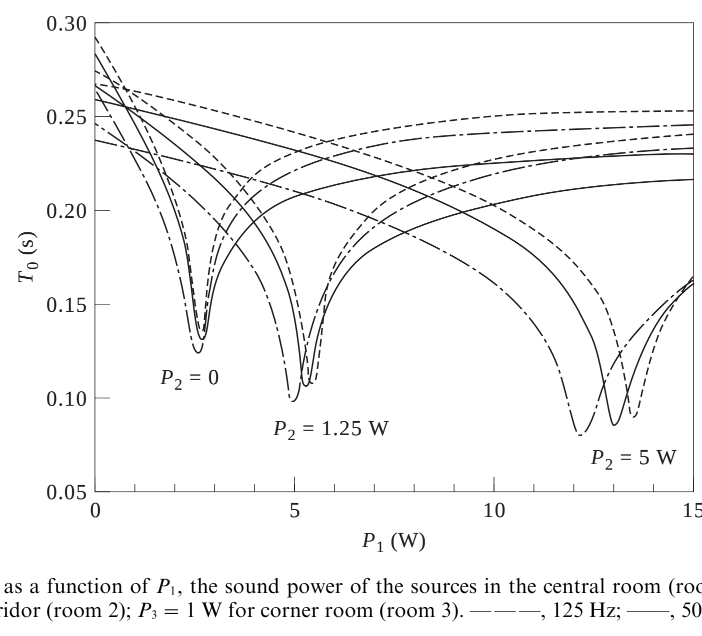

Figure 14. T_0 as a function of P_1, the sound power of the sources in the central room (room 1); $P_2 = 0$, 1·25 and 5 W for corridor (room 2); $P_3 = 1$ W for corner room (room 3). ———, 125 Hz; ——, 500 Hz; –·–·–, 2 kHz.

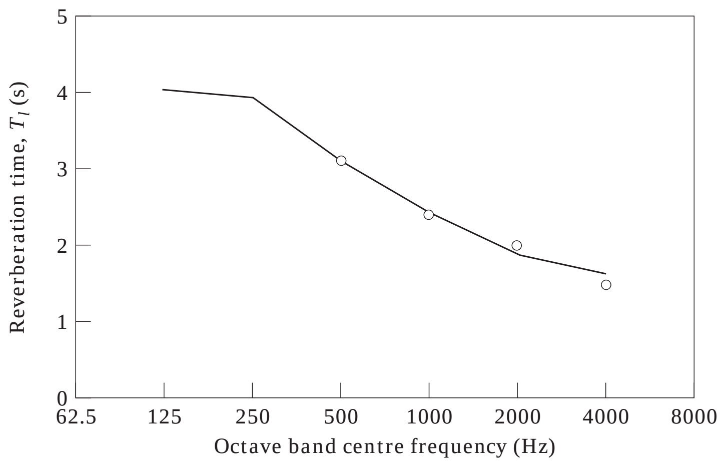

Figure 15. The reverberation time T_l as a function of frequency. ——, Results calculated by the approximate model; ○, measured results.

of which are not fully known. In other situations, in comparison with the general approach, the advantages of the approximate model could become a disadvantage, as it does not incorporate different boundary conditions (because of the assumption that the interior space is infinite). It is also *a priori* limited to cases with a regular distribution of sound sources throughout the whole space; all rooms of the same type, e.g., the central rooms, have sound sources of the same sound power. All the sound sources are switched off simultaneously at $t = 0$. Thus the sound energy densities in all rooms of the same type are always the same.

For the large interiors considered, with a repeatable pattern and with strong coupling between the rooms, the rate of sound decay during the late stage is the same for all rooms and does not depend upon the distribution of sound sources. In this case the estimation of the value of the reverberation time T_l (referred to as the late stage of sound decay) could be deduced with sufficient accuracy by using the approximate model

The dependence of the reverberation time T_l on frequency is shown in Figure 15. These values are quite close to those obtained experimentally.

6. CONCLUSIONS

For interiors with a sufficiently large number of coupled rooms, the effect of coupling should be taken into account even though the coupling areas between individual rooms are relatively great. In such cases, the coupling between rooms mainly influences the sound decay during the early stage of the process, a point confirmed by experimental results. The duration of the early stage of sound decay depends on the distribution of sound sources throughout the space and the ratios of their powers. There exist optimal distributions of sound sources leading to minimization of coupling effects. The most pronounced coupling phenomena can be observed for the cases in which both the sound sources and the equivalent absorption areas of the rooms are asymmetrically distributed in space.

For an interior space with a large floor area but a relatively small height and with a repetitive architectural pattern, the simple approximate model of coupled rooms can be introduced, which gives a good agreement with the general model in a substantial part of the space and can be essentially useful in the optimization of the distribution of the sound sources.

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